Toward Understanding the Visual System: the emergence of simple cells in V1

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in collaboration with
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FIAS
Talk Contents

- Visual System Biology
- Models of Edge Detectors in V1
  - Weak Non-Linearities
  - Strong Non-Linearities
Visual System Biology
Areas in the Visual System
Hierarchy of Visual Areas
Figure 24. Onset latencies. Cumulative distributions of visually evoked onset response latencies in the LGNd, striate and extrastriate areas as labeled. Percentile of cells that have begun to respond is plotted as a function of time from stimulus presentation. From Schmolesky et al. (1998).
Cortical Layers

Nissl stained section of the visual cortex to show the border between area 17 (V1) and area 18 (V2).

Nissl stain

Cytochrome oxidase

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Cortical Maps (V1)

Orientation Columns

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Cortical Maps (V1)

Orientation Columns
p31

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ocular dominance stripes
Hubel & Wiesel 1960’s
Tilt Aftereffect (TAE)
Models of Edge Detectors in V1
Localized Edge Detector

**Input**: \( \vec{x} \)

**Output**: \( z(y) = \frac{e^y}{1+e^y} \)

"Net input"

\[
y_i = \sum_j w_{ij} x_j - b_i
\]

weights

threshold

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Localised Edge Detector

Input $\vec{x}$

Output $z(y) = \frac{e^y}{1+e^y}$

"Net input"

$y_i = \sum_j w_{ij} x_j - b_i$

weights

threshold

... to be learnt
Weak Non-Linearities – Sparseness
Statistics of Edges in Images

Picture 3

Line

Random
A Sparse Forward Model

Data examples $\vec{x}$

$p = \text{probability each line present}$

$p^2 = \text{probability 2 lines co-occurring}$

Weights $W^{bu}$
A Sparse Forward Model

Data examples $\vec{x}$

$p = \text{probability each line present}$

$p^2 = \text{probability 2 lines co-occurring}$

Weights $W^{bu}$
A Sparse Forward Model

Activation update

\[ \Delta z_i = g\left(\overline{w}_{bu}^i \cdot \overline{x} + \overline{w}_{lat}^i \cdot \overline{z} - b_i\right) \]

\[ z_i \rightarrow 0, 1 \]

Thresholds

\[ \Delta b_i \approx z_i - p \]

Lateral weights

\[ \Delta w_{ij}^{lat} \approx p^2 - z_i z_j \]

BU weights

\[ \Delta \overline{w}_i \approx z_i (\overline{x} - \overline{w}_i) \]
A Sparse Forward Model

Data examples $\vec{x}$

Weights $W^{bu}$
A Sparse Forward Model

Data examples $\vec{x}$

Weights $W^{bu}$
A Sparse Forward Model

Data examples $\tilde{\mathbf{x}}$

Weights $W^{bu}$

... should be:
Entropy Max and Generative Model

\[ H(\tilde{z}(\tilde{y}(W^{bu}))) = \text{MAX!} \]

\[ (\tilde{x} - W^{td} \tilde{z})^2 = \text{MIN!} \]
Entropy Max

\[ p^+(x) \xrightarrow{W_{bu}} z(y) \]

\[ p^-(y) \]

\[ p^+(y) \]

\[ p^+(z) \]

\[ p^- (z) \]

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Generative Model

\[ p(x) - p(y) - z(y) - p^-(z) \]

\[ p^+(x) - p^-(x) \]

\[ W^{td} \]
Reconstruction error
\[ \tilde{x} = \tilde{x} - W^{td} \tilde{y} \]

An energy function
\[ E(W, \tilde{y}) = \frac{1}{2} \tilde{x}^2 \]

Activation update
\[ \Delta \tilde{y} \approx -\frac{\partial E}{\partial \tilde{y}} = W^{bu} \tilde{x} \]

Learning
\[ \Delta W^{td} \approx -\frac{\partial E}{\partial W^{td}} = \tilde{x} \tilde{y}^T \]
\[ W^{bu} = (W^{td})^T \]
Reconstruction error
\[ \tilde{x} = \bar{x} - W^{td} \bar{y} \]

An energy function
\[ E(W, \bar{y}) = \frac{1}{2} \tilde{x}^2 \]

Activation update
\[ \Delta \bar{y} \approx -\frac{\partial E}{\partial \bar{y}} = W^{bu} \tilde{x} \]

Learning
\[ \Delta W^{td} \approx -\frac{\partial E}{\partial W^{td}} = \tilde{x} \bar{y}^T \]

\[ W^{bu} = (W^{td})^T \]

Linear model does not make any sense!
Sparse Distribution Requested

\[ y = W x \]

\[ z = g(y) \]

\[ x = W^{td} z \]

\[ g(y) = \frac{e^{ay-b}}{1 + e^{ay-b}} \]

- intrinsic plasticity parameters \( a, b \)
- medium time course of parameter adaptation
Adaptation of $a$ and $b$

\[
\frac{\partial}{\partial t} a_i = -\eta_a \frac{\partial}{\partial a_i} KL(f_z(z) \| \frac{1}{\mu} e^{-z/\mu}) \\
= ... \\
= \eta_a \left( \frac{1}{a_i} + y_i - 2y_i z_i - \frac{1}{\mu} y_i z_i + \frac{1}{\mu} y_i z_i^2 \right)
\]

\[
\frac{\partial}{\partial t} b_i = -\eta_b \frac{\partial}{\partial b_i} KL(f_z(z) \| \frac{1}{\mu} e^{-z/\mu}) \\
= ... \\
= \eta_b \left( 1 - 2z_i - \frac{1}{\mu} z_i + \frac{1}{\mu} z_i^2 \right)
\]

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TAE – how $a$ and $b$ adapt

The diagram illustrates the decrease of parameter $a$ and $b$ as a function of orientation difference. The x-axis represents the orientation difference, ranging from $-90$ to $90$ degrees. The y-axis represents the decrease of parameter, ranging from $-1$ to $0$. Two curves are shown, one for $a$ adapt (black line) and one for $b$ adapt (blue line).
Strong Non-Linearities – Competition
Hierarchical Linear Model

\[ \vec{y}_2 = W^2 \vec{y}_1 \]

\[ = W^2 W^1 \vec{x} \]
Hierarchical Linear Model

\[ \vec{y}_2 = W^2 \vec{y}_1 \]
\[ = W^2 W^1 \vec{x} \]
\[ = \tilde{W} \vec{x} \]

makes no sense
Non-Linearities

Retinal input

V1 activation

linear  sparse  competitive  winner
Non-Linearities

Retinal input

V1 activation

linear
Non-Linearities

Retinal input

V1 activation

linear  sparse  competitive  winner
Setting up Competition

Weight profile

Activation update

\[ \tilde{z}_i(t + 1) = g(\vec{w}_i \cdot \tilde{z}) \]
Functions of Competitive Non-Linearity

- noise removal
- categorization/discretization of continuous data
- computing invariances (complex cells)
- segmentation/binding – via (de-) synchronization
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Other evidence for competition
- retrieve stimulus parameter with max likelihood
- contrast-invariant orientation tuning curve widths
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Learning the Lateral Weights

Activation initialization
\[ \tilde{z}_i(t^0) = z_i(t^0) \]

Activation update
\[ \tilde{z}_i(t + 1) = f(\tilde{w}_i^{\text{lat}} \cdot \tilde{z}(t)) \]

Lateral weights
\[ \Delta w_{ij}^{\text{lat}} \approx (\tilde{z}_i(t_{\text{end}}) - \tilde{z}_i(t_{\text{end}})) \cdot \tilde{z}_j(t_{\text{end}} - 1) \]
$W^{lat}$ and $W^{bu}$

**lateral RF**

**BU RF’s 0**

**BU RF’s 2**

**BU RF’s 1**

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Results

Lateral weights

Orientation tuning

spatial frequency — orientation

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Results

Lateral weights

Orientation tuning

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Outlook ...

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