Rules for Information Maximization in Spiking Neurons Using Intrinsic Plasticity

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Synopsis

- Neurons in various sensory modalities transform the stimuli into series of action potentials

- The mutual information between input and output distributions should be maximized

- Biological Evidence: V1 neurons in cat and macaque respond with an approximately exponential distribution of firing rates
Synopsis

- **Intrinsic plasticity** is the persistent modification of a neuron’s intrinsic electrical properties by neuronal or synaptic activity.

- It has been hypothesized that intrinsic plasticity plays a distinct role in firing rate homeostasis and leads to an approximately exponential distribution of firing rate.

- This work derives two gradient based intrinsic plasticity rules with both rules leading to information maximization:
  - **Rule 1**: Direct maximization of MI
  - **Rule 2**: Minimize the Kullback-Leibler divergence between OP distribution and a desired exponential distribution.

- IP is achieved by adapting the gain function of a neuron to its input distribution.
Outline

- Intrinsic plasticity in biology
- Computational theory and learning rules
  - Neuron model
  - IP Rule 1
  - IP Rule 2
- Simulation Results
- Conclusion
Intrinsic Plasticity in Biology

- Trace eyelid conditioning task (*Disterhoft et. al.*)

- Recordings from CA1 pyramidal cells showed a transient (~1-3 days) increase in excitability

Figure from: W. Zhang, D. J. Linden. The other side of engram: Experience-driven changes in neuronal intrinsic excitability. *Nat. Rev. Neurosci.*, Vol 4, pp. 885-900
Neuron Model

- Stochastically spiking neuron with refractoriness (Toyozumi et. al. )

\[ \tau_m = 10 \text{ ms}, \ \tau_{\text{refr}} = 10 \text{ ms}, \ \tau_{\text{abs}} = 3 \text{ ms} \]

Membrane Potential

\[ u_i(t^k) = u_r + \sum_{j=1}^{N} \sum_{n=1}^{N} w_{ij} \epsilon(t^k - t^n) x_j^n \]

Refractoriness

\[ R_i(t) = \frac{(t - \hat{t}_i - \tau_{\text{abs}})^2}{\tau_{\text{refr}}^2 + (t - \hat{t}_i - \tau_{\text{abs}})^2} \Theta(t - \hat{t}_i - \tau_{\text{abs}}) \]

Probability of Spiking

\[ \rho_i^k = 1 - e^{-g(u_i(t^k))R_i(t^k)\Delta t} \approx g(u_i(t^k))R_i(t^k)\Delta t \]

Gain Function(s)

- with refractoriness

\[ g(u) = r_0 \log \left(1 + e^{\frac{u-u_0}{u_\alpha}}\right) \]

- without refractoriness

\[ \tilde{g}(u) = \left(\frac{1}{g_{\text{max}}} + \frac{1}{g(u)}\right)^{-1} \]
**IP Rule 1: Direct maximization of mutual information**

- **Key Idea:** To maximize
  \[ I(y, x) = H(y) - H(y|x) \]  
  \[ (1) \]

- Equivalent to maximizing
  \[ H(y) = -E[\log f_y(y)] = - \int_{-\infty}^{\infty} f_y(y) \log f_y(y) \, dy \]  
  \[ (2) \]

- Where,
  \[ f_y(y) = \frac{f_u(u)}{\frac{\partial y}{\partial u}} \]  
  \[ (3) \]

- Substituting (3) into (2) we get the term for maximization as:
  \[ H(y) = E\left[\log \frac{\partial y}{\partial u}\right] - E[\log f_u(u)] \]  
  \[ (4) \]

- Learning rule consist of a set of update equations for various parameters \( \varphi \) of the gain function
  \[ \Delta \varphi = \eta_{MI} \frac{\partial H(y)}{\partial \varphi} = \eta_{MI} \frac{\partial}{\partial \varphi} \left( \log \frac{\partial y}{\partial u} \right) \]
  \[ = \eta_{MI} \left( \frac{\partial y}{\partial u} \right)^{-1} \frac{\partial}{\partial \varphi} \left( \frac{\partial y}{\partial u} \right) \]  
  \[ (5) \]
Rule 1 Update Equations

\[ u_0 \leftarrow u_0 + \Delta u_0 \]

\[ \Delta u_0 = -\frac{\eta_{MI}}{u_\alpha} e^{-\frac{g}{r_0}} \]

\[ u_\alpha \leftarrow u_\alpha + \Delta u_\alpha \]

\[ \Delta u_\alpha = -\frac{\eta_{MI}}{u_\alpha} \left( 1 + \frac{u - u_0}{u_\alpha} e^{-\frac{g}{r_0}} \right) \]

Note that similar analysis for the term \( r_0 \) leads to an update rule which will cause the value of \( r_0 \) to increase without any constraint, hence it is not included in the set of update rules.
IP Rule 2: Minimizing the KL Divergence

- Key Idea: Minimize the KLD between $f_y(y)$ and the optimal exponential distribution $f_{\text{exp}}(y)$
- KL Divergence is defined as:

$$D \equiv d(f_y||f_{\text{exp}}) = \int f_y(y) \log \left( \frac{f_y(y)}{\frac{1}{\mu} e^{-\frac{y}{\mu}}} \right) dy$$

$$= \int f_y(y) \log(f_y(y)) dy - \int f_y(y) \left( -\frac{y}{\mu} - \log \mu \right) dy$$

or

$$D \equiv d(f_y||f_{\text{exp}}) = -H(y) + \frac{1}{\mu} E(y) + \log \mu$$

- Learning rule consist of a set of update equations for various parameters $\varphi$ of the gain function

$$\frac{\partial D}{\partial \varphi} = \frac{\partial d(f_y||f_{\text{exp}})}{\partial \varphi} = -\frac{\partial H(y)}{\partial \varphi} + \frac{1}{\mu} \frac{\partial y}{\partial \varphi}$$

$$= E \left[ -\frac{\partial}{\partial \varphi} \left( \log \left( \frac{\partial y}{\partial u} \right) \right) + \frac{1}{\mu} \frac{\partial y}{\partial \varphi} \right]$$
Rule 2 Update Equations

\[ r_0 \leftarrow r_0 + \Delta r_0 \]
\[ \Delta r_0 = \frac{\eta_{IP}}{r_0} \left( 1 - \frac{g}{\mu} \right) \]

\[ u_0 \leftarrow u_0 + \Delta u_0 \]
\[ \Delta u_0 = \frac{\eta_{IP}}{u_\alpha} \left[ \left( 1 + \frac{r_0}{\mu} \right) \left( 1 - e^{-\frac{g}{r_0}} \right) - 1 \right] \]

\[ u_\alpha \leftarrow u_\alpha + \Delta u_\alpha \]
\[ \Delta u_\alpha = \frac{\eta_{IP}}{u_\alpha} \left[ -1 + \frac{u - u_0}{u_\alpha} \left( 1 + \frac{r_0}{\mu} \right) \left( 1 - e^{-\frac{g}{r_0}} \right) - 1 \right] \]
Results: Performance of IP Rule 1 (MI Max)

- Inputs: 100 Spike trains, Gaussian Dist. (mean = 25 Hz. SD = 5 Hz)
- $\eta_{\text{MI}} = 10^{-3}$, $T = 10$ min.

\[ \Delta u_0 = -\frac{\eta_{\text{MI}}}{u_\alpha} e^{-\frac{g}{r_0}} \]
\[ \Delta u_\alpha = -\frac{\eta_{\text{MI}}}{u_\alpha} \left( 1 + \frac{u - u_0}{u_\alpha} e^{-\frac{g}{r_0}} \right) \]
Results: Performance of IP Rule 2 (KLD Min)

- Inputs: 100 Spike trains, Gaussian Dist. (mean = 30 Hz, SD = 5 Hz)
- $\eta_{IP} = 10^{-5}$, $\mu = 1.5$ Hz, $T = 16.67$ min.

$\Delta r_0 = \frac{\eta_{IP}}{r_0} \left( 1 - \frac{g}{\mu} \right)$

$\Delta u_0 = \frac{\eta_{IP}}{u_\alpha} \left[ \left( 1 + \frac{r_0}{\mu} \right) \left( 1 - e^{-\frac{g}{r_0}} \right) - 1 \right]$
Results: Convergence of Rule 2

- $\eta_{IP} = 10^{-3}$, $\mu = 1.5$ Hz
- Evolution of trajectories from 3 different initial conditions
Results: Phase Plots

- $\eta_{IP} = 10^{-3}$, $\mu = 1.5$ Hz

- Pair-wise phase-portraits indicating the flow field, while keeping the third parameter constant
Results: Behavior for various IP dist.

- $\eta_{IP} = 10^{-3}$, $\mu = 1.5$ Hz
- Gaussian IP Dist: Mean = 30 Hz, SD = 8Hz
- Uniform IP Dist: Drawn from [0,60] Hz
- Exponential IP Dist: Scale parameter, $\beta = 30$ Hz
Results: Adaptation to sensory deprivation

- **First half:** Gaussian (mean = 30 Hz, SD = 5 Hz)
- **Second half:** Gaussian (mean = 5 Hz, SD = 1 Hz)
Conclusions

- Two simple gradient based rules for IP are presented
- First rule used direct maximization of MI
- Second rule minimizes the KLD between $f_y(y)$ and the optimal exponential distribution $f_{\exp}(y)$
- Adapt the gain function of a model neuron according to sensory stimuli
- Valid approach for neuron models which have continuous and differentiable gain functions
- Works for several different input distributions
- Leads to exponential output distribution, firing rate homeostasis, and adapts to sensory deprivation
THANK YOU
References


